



K21U 4552

Reg. No. :

Name :

Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time : 3 Hours

Max. Marks : 48

PART – A
(Short Answer)

Answer any 4 questions. Each question carries 1 mark.

1. Define abelian group with an example.
2. Is \mathbb{Z}^* under division a binary operation. Justify.
3. Every infinite order cyclic group is isomorphic to
4. What is the order of alternating group A_n ?
5. State Lagrange's theorem.

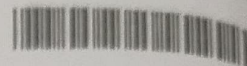
(4×1=4)

PART – B
(Short Essay)

Answer any eight questions. Each question carries 2 marks.

1. In a group G with binary operation $*$, prove that there is only one element e in G such that $e * x = x * e = x, \forall x \in G$.
2. Prove that $(\mathbb{Q}^+, *)$, where $*$ is defined by $a * b = \frac{ab}{2}$; $a, b \in \mathbb{Q}^+$ is a group.
3. For sets H and K , Let $H \cap K = \{x/x \in H \text{ and } x \in K\}$, show that if H and K are subgroups of a group G , then $H \cap K$ is also a subgroup of G .

P.T.O.



9. Prove that the order of an element of a finite group divides the order of group
10. Explain the elements of group S_3 .
11. Find the order of $(14)(3578)$ in S_8 .
12. Prove that every permutation σ of a finite set is a product of disjoint cycles.
13. Determine the permutation $(18)(364)(57)$ in S_8 is odd or even.
14. State fundamental homomorphism theorem.
15. Find the order of $\mathbb{Z}_6 / \langle 3 \rangle$.
16. Let $\phi : G \rightarrow G'$ be a group homomorphism. Prove that $\text{Ker}\phi$ is a subgroup of G .

(8x)

PART – C

(Essay)

Answer any four questions. Each question carries 4 marks.

17. Prove that subgroup of a cyclic group is cyclic.
18. Let G be a group and $a \in G$. Prove that $H = \{a^n / n \in \mathbb{Z}\}$ is the smallest subgroup of G that contains a .
19. Determine whether the set of all $n \times n$ matrices with determinant -1 is a subgroup of G .
20. Let A be a non-empty set. Prove that S_A , the collection of all permutations A is group under permutation multiplication.
21. Define rings. Prove that $(\mathbb{Z}_n, +_n, \times_n)$ is a ring.
22. Prove that every group is isomorphic to a group of permutations.
23. Prove that $\gamma : \mathbb{Z} \rightarrow \mathbb{Z}_n$; where $\gamma(m) = r$; where r is the remainder when m divided by n is a homomorphism.

(4x)

PART – D
(Long Essay)

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that every integral domain is a field.
25. Let H be a subgroup of G , then prove that the left coset multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if H is a normal subgroup of G .
26. a) Find all cosets of the subgroup $\langle 2 \rangle$ of \mathbb{Z}_{12} .
b) Prove that every group of prime order is cyclic.
27. Let G be a cyclic group with n elements generated by a . Let $b \in G$ and $b = a^s$, then prove that b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where d is the gcd of n and s . (2×6=12)
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